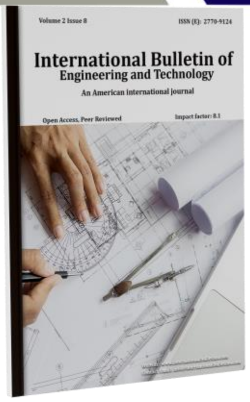

ON THE STABILITY OF THE GALERKIN METHOD FOR SOLVING THE PROBLEM OF DETERMINING THE WARM-MOISTURE STATE OF RAW COTTON

A.Z.Mamatov [1]
J.T. Raxmanov [2]
N.O.Sulaymanova [3]
[1] Professor of the Tashkent Institute of Textile and Light Industry, Tashkent city, Uzbekistan
[2] Basic doctorant of Gulistan State University, Syrdarya region, town of Gulistan, Uzbekistan
[3] Intern teacher of Guliston State Pedagogical Institute
email: jamshidbekmrm2012@gmail.com
https://doi.org/10.5281/zenodo.8037373

ANNOTATION

The article considers a boundary value problem for systems consisting of two differential equations of parabolic type to determine the heat-moist state of raw cotton in a direct-flow drum dryer. An approximate solution of the Galerkin method for the problem under consideration is constructed. The stability of the Galerkin method for the approximate solution of the problem under consideration is established under the condition of strongly minimal coordinate systems.

Key words: mathematical model, algorithm, temperature, coordinate system, monotonicity, stability, strong minimality,.

INTRODUCTION. In the process of drying wet raw cotton, a complex non-stationary heat-mass transfer process occurs, which determines the external and internal states. External processes are characterized by mass transfer from the surface of raw cotton to the environment and heat transfer between the fiber and the environment. Important for maintaining the quality of the fiber and seeds during drying is the rate of distribution of heat and moisture of raw cotton through the drum dryer. [1-7].

In this paper, we study the problem of determining the heat-moist state of raw cotton during drying in a dryer. The speed of movement of raw cotton v is assumed to be constant and the same in the section of the installation. Let us assume that there is a convective heat exchange according to Newton's law between raw cotton and air. Then the warm-moist state of raw cotton in the dryer can be determined from the initial-boundary value problem in the form:

$$\begin{cases} c\rho\dfrac{\partial T}{\partial \tau} = \lambda\dfrac{\partial^2 T}{\partial x^2} - \vartheta c\rho\dfrac{\partial T}{\partial x} - \alpha(T - T_B) + \varepsilon\,\rho\, r_{21}\dfrac{\partial U}{\partial \tau} \\ c_m\rho\dfrac{\partial U}{\partial \tau} = \lambda_m\dfrac{\partial^2 U}{\partial x^2} + \lambda_m\delta\dfrac{\partial^2 T}{\partial x^2} - \beta(U - U_B) - \vartheta c_m\rho\dfrac{\partial U}{\partial x} \end{cases} \quad (1)$$

with initial

$$T(x,0)=T_0,\quad U(x,0)=U_0 \quad (2)$$

and boundary conditions

$$\left.\frac{\partial U}{\partial x}\right|_{x=0}=0 \quad , \quad \left.\frac{\partial U}{\partial x}\right|_{x=l}=0$$

$$-\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0}=\alpha_1(T-T_B)\ , \qquad \lambda\left.\frac{\partial T}{\partial x}\right|_{x=l}=0 \tag{3}$$

where T, T_w – respectively, the temperature of raw cotton, drying agent; U, U_w – respectively, the moisture content of raw cotton and air; $c, \lambda, \rho, \vartheta$ - respectively, the heat capacity, thermal conductivity, density and speed of movement of raw cotton, α - volumetric heat transfer coefficient between raw cotton and air; ε - phase transformation coefficient, r_{21} - heat of vaporization, τ - drying time, ℓ - installation length.

SOLUTION METHOD. To solve this problem, we use the Bubnov-Galerkin projection method. We introduce two sets of basis functions and denote them by $\{\varphi_i\}$, $\{\psi_i\}$. From the elements of the basis functions, we require that they have a second derivative with respect to the spatial variables.

We will look for approximate solutions of the system in the form [8-16]

$$T=\sum_{k=1}^{N}c_k(\tau)\cdot\varphi_k(x); \quad U=\sum_{ki=1}^{N}d_k(\tau)\cdot\psi_k(x) \tag{4}$$

where the coefficients $c_k(\tau)$, $d_k(\tau)$ are determined from the system of equations

$$\begin{cases} Q_n\cdot\dfrac{dC_n(\tau)}{d\tau}+P_nC_n(\tau)+G_nD_n(\tau)=F_{1n}(\tau) \\ \tilde{Q}_n\cdot\dfrac{dD_n(\tau)}{d\tau}+\tilde{P}_nD_n(\tau)+\tilde{G}_nC_n(\tau)=F_{2n}(\tau) \\ Q_nC_n(0)=F_{10} \\ \tilde{Q}_nD_n(0)=F_{20} \end{cases} \tag{5}$$

where $Q_n=(\alpha_{ik})$, $P_n=(\beta_{ik})$, $G_n=(\gamma_{ik})$, $\tilde{Q}_n=(\tilde{\alpha}_{ik})$ $\tilde{P}_n=(\tilde{\beta}_{ik})$ и $\tilde{G}_n=(\tilde{\gamma}_{ik})$ square matrices of size (NxN);

$C_n(\tau)=(c_1(\tau),c_2(\tau),...,c_n(\tau))^T$, $D_n(\tau)=(d_1(\tau), d_2(\tau),...,d_n(\tau))^T$ – desired vectors;
$F_{1n}(\tau)=(f_{11}(\tau),f_{12}(\tau),...,f_{1n}(\tau))^T$, $F_{2n}(\tau)=(f_{21}(\tau),f_{22}(\tau),...,f_{2n}(\tau))^T$ given vectors.

Elements of vectors $F_{10}(\tau)=(f_{01}(\tau),f_{02}(\tau),...,f_{0n}(\tau))^T$ and $F_{20}(\tau)=\left(\tilde{f}_{01}(\tau),\ \tilde{f}_{02}(\tau),\ \dots,\ \tilde{f}_{0n}(\tau)\right)^T$ are determined from the relations:

$$f_{0i}=(T_0,\varphi_i(x)), \quad \tilde{f}_{0i}=(U_0,\psi_i(x)) \tag{6}$$

As is known, from the theory of ordinary differential equations, if the matrices are non-degenerate and positive-definite, the system (5) composed of the coefficients of the system has a unique solution.

Selecting the basis functions and constructing implicit difference schemes on the interval *[0;l]*, we obtain a system of algebraic equations:

$$\begin{cases} (Q_n+\Delta\tau P_n)\cdot C_n^{l+1}(\tau)+G_nD_n^{l+1}(\tau)=F_{1n}^l(\tau)-Q_nC_n^l(\tau) \\ \widetilde{G}_nC_n^{l+1}(\tau)+(Q_n+\Delta\tau\widetilde{P}_n)\cdot D_n^{l+1}(\tau)=F_{2n}^l(\tau)-Q_nD_n^l(\tau) \\ Q_nC_n^0(\tau)=F_{10} \\ Q_nD_n^0(\tau)=F_{20} \qquad l=0,1,2,3,\ldots,M \end{cases} \quad (7)$$

The system of algebraic equations (7) is solved by the Gauss method. The found value $C_n^l(\tau)$, $D_n^l(\tau)$ substituting (4) and we find the temperature and moisture content of raw cotton during the drying process.

We are now exploring question of the stability of problem (5). Let us assume that the coordinate systems $\{\varphi_i(x)\}$, $\{\psi_i(x)\}$ are strongly minimal in the space $L_2(\Omega)$ i.e. there is a constant independent of n such that $0<q<q_i^n$, where q_i^n are the eigenvalues of the matrix

$$Q_n=\{(\varphi_k,\varphi_j)_{L_2}\}_{k,\ j=1}^n \qquad \widetilde{Q}_n=\{(\psi_k,\psi_j)_{L_2}\}_{k,\ j=1}^n$$

Let us assume that instead of the Galerkin system (5) we solve the "perturbed" system

$$\begin{cases} (Q_n+\Gamma_n)\cdot\dfrac{dC_n(\tau)}{d\tau}+(P_n+\Gamma_n^1)C_n(\tau)+(G_n+\Gamma_n'')D_n(\tau)=F_{1n}(\tau)+\delta_n^1 \\ (\tilde{Q}_n+\tilde{\Gamma}_n)\cdot\dfrac{dD_n(\tau)}{d\tau}+(\tilde{P}_n+\tilde{\Gamma}_n^1)D_n(\tau)+(\tilde{G}_n+\tilde{\Gamma}_n'')C_n(\tau)=F_{2n}(\tau)+\tilde{\delta}_n^1 \\ (Q_n+\Gamma_n^0)\tilde{C}_n(0)=F_{10}+\varepsilon_{10} \\ (\tilde{Q}_n+\tilde{\Gamma}_n^0)\tilde{D}_n(0)=F_{20}+\varepsilon_{20} \end{cases} \quad (8)$$

where $\tilde{C}_n(\tau)$, $D_n^l(\tau)$ - solution of the perturbed problem.

A Galerkin process for a given problem is called stable if there exist positive constants p_i independent of n such that for sufficiently small matrix norms $\|\Gamma_n^0\|$, $\|\Gamma_n\|$, $\|\Gamma_n'\|$ and for vector norms are performed inequalities

$$\|\tilde{C}_n(\tau)-C_n(\tau)\|_{E_n}\le p_0\|\varepsilon_0\|+p_1\|\varepsilon_n\|+p_2\|\Gamma_n^0\|+p_3\|\Gamma_n\|+p_4\|\Gamma_n'\|$$

$$\|\tilde{d}_n(\tau)-d_n(\tau)\|_{E_n}\le\tilde{p}_0\|\tilde{\varepsilon}_0\|+\tilde{p}_1\|\tilde{\varepsilon}_n\|+\tilde{p}_2\|\tilde{\Gamma}_n^0\|+\tilde{p}_3\|\tilde{\Gamma}_n\|+\tilde{p}_4\|\Gamma_n'\| \quad (9)$$

An approximate solution $U(r,\tau)=(\theta_{1n},\theta_{2n}(r,\tau))^T$ is called stable in the space $L_2(\Omega)$, if an inequality similar to (9) holds for the difference, where $\tilde{U}(r,\tau)=(\tilde{\theta}_{1n}(r,\tau),\tilde{\theta}_{2n}(r,\tau))^T$, $\quad\tilde{\theta}_{in}(r,\tau)=\sum_{k=1}^n\tilde{a}_{iK}(\tau)\cdot\varphi_K(r)$

Similarly, as in the previous paragraph, multiplying each of the equations of system (5) by the corresponding $a_{1i}(\tau)$, $a_{2i}(\tau)$ and making similar calculations, one can establish a continuous dependence of the approximate solutions on the initial data and right-hand sides in the form:

$$\begin{cases} \|\theta_{1n}\|_2^2 + \int_0^\tau \|\nabla\theta_{1n}\|_2^2 \le p_1\left[u_B^2 + \|g_1(r)\|_2^2 + \|g_2(r)\|_2^2 + \int_0^\tau f_1^2(\tau)d\tau + \int_0^\tau f_2^2(\tau)d\tau\right] \\ \|\theta_{2n}\|_2^2 + \int_0^\tau \|\nabla\theta_{2n}\|_2^2 \le p_2\left[u_B^2 + \|g_2(r)\|_2^2 + \|g_1(r)\|_2^2 + \int_0^\tau f_1^2(\tau)d\tau + \int_0^\tau f_2^2(\tau)d\tau\right] \end{cases} \quad (10)$$

In addition, taking into account the continuity of the given functions and using the mean value theorem, we can estimate

$$\|\theta_{1n}\|^2 + \int_0^\tau \|\nabla\theta_{1n}\|^2 \le M_1, \qquad \|\theta_{2n}\|^2 + \int_0^\tau \|\nabla\theta_{2n}\|^2 \le M_2$$

Hence, using the strong minimality of the basis functions in $L_2(\Omega)$, we can obtain the following inequalities:

$$\|G_n(\tau)\|_{E_n}^2 \le \frac{1}{q}\|U(r,\tau)\|_{L_2(\Omega)}^2 \le M \;, \quad \int_0^\tau \|\dot{G}_n(\tau)\|_{E_n}^2 d\tau' \le \frac{1}{q}\int_0^\tau \left\|\frac{\partial U(r,\tau)}{\partial\tau}\right\|_{L_2(\Omega)}^2 d\tau' \le K, \quad (11)$$

where $G_n(\tau) = (a_{1i}(\tau),\; a_{2i}(\tau))^T$

Let the allowed errors $\Gamma_n, \Gamma'_n, \Gamma_n, \Gamma'_n$ are as follows

$$\|\Gamma_n\| \le e_1 q, \quad \|\Gamma'_n\| \le e_2 q; \quad 0 \le e_i \le 1, \quad q > 0. \quad (12)$$

Denote by $Z_n(\tau) = \tilde{G}_n(\tau) - G_n(\tau)$.

Let us subtract the systems of equations (5) from the system of equations (7). The resulting equation is scalarly multiplied by $\dot{Z}_n(\tau)$, i.e.

$$\frac{1}{2}\frac{d}{d\tau}((Q_n + \Gamma_n)Z_n, Z_n) + ((P_n + \Gamma'_n)Z_n, \dot{Z}_n) = (\varepsilon_n, \dot{Z}_n) + (\Phi_n, \dot{Z}_n) \quad (13)$$

where $\Phi_n(\tau) = -\Gamma_n \cdot \dot{G}(\tau) - \Gamma'_n \; G_n(\tau)$.

Since the matrix is positive definite, then

$$((P_n + \Gamma'_n)Z_n, \dot{Z}_n)_n \ge 0.$$

Then, estimating the terms of the right side of the equality

$$|(\varepsilon_n, \dot{Z}_n)_{E_n}| \le \frac{1}{2\varepsilon_1}\|\varepsilon_n\|^2 + \frac{1}{2}\varepsilon_1\|Z_n\|^2$$

and $$|(\Phi_n(\tau), \dot{Z}_n)| \le \frac{1}{2\varepsilon_1}\|\Phi_n(\tau)\|^2 + \frac{1}{2}\varepsilon_1\|Z_n\|^2$$

we obtain

$$\frac{1}{2}\frac{d}{d\tau}((Q_n + \Gamma_n)Z_n, Z_n) \le \varepsilon_1\|Z_n\|^2 + c_1(\|\varepsilon_n\|^2 + \|\Phi_n(\tau)\|^2)$$

We integrate the last inequalities over τ. Paying attention to the inequality

$$\|Z_n\|_{E_n}^2 \le \frac{1}{2}\|\tilde{U}-U\|_{L_2}^2$$

We obtain

$$((Q_n+\Gamma_n)Z_n, Z_n)_{E_n} \le 2\varepsilon_1\int_0^\tau \|\tilde{U}-U\|_{L_2}^2 d\tau + c_1\int_0^\tau \left(\|\varepsilon_n\|^2 + \|\Phi_n(\tau)\|_{L_2}^2\right)d\tau + ((Q_n+\Gamma_n)Z_n(0), Z_n(0))_{E_n}$$

On the other hand,

$$((Q_n+\Gamma_n)Z_n, Z_n)_{E_n} \ge (Q_n Z_n, Z_n) - e_1 q\|Z_n\|_{E_n}^2 \ge (1-e_1)\|\tilde{U}_n - U_n\|_{L_2}^2,$$

$$((Q_n+\Gamma_n)Z_n(0), Z_n(0))_{E_n} \le (Q_n Z_n(0), Z_n(0))_{E_n} + e_1 q\|Z_n\|_{E_n}^2 \le c_2(1+e_1)\|\tilde{U}(r,0)-U(r,0)\|_{L_2(\Omega)}^2$$

and by virtue of estimate (12)

$$\int_0^\tau \|\Phi_n(\tau)\|_{E_n}^2 d\tau' \le 2M\int \|\Gamma_n\|^2 d\tau' + 2K\int_0^\tau \|\Gamma_n'\|^2 d\tau' \le 2MT\|\Gamma_n\|^2 + 2KT\|\Gamma_n'\|_{E_n}^2$$

We denote by

$$\int_0^\tau \|\tilde{U}_n(r,\tau) - U_n(r,\tau)\|_{L_2}^2 d\tau = y(\tau)$$

$$F_n(\tau) = c_2(1+e_1)\|\tilde{U}_n(r,0)-U_n(r,0)\|_2^2 + c_1\|\varepsilon_n\|_{E_n}^2 + 2MT\|\Gamma_n\|_{E_n}^2 + 2KT\|\Gamma_n'\|_{E_n}^2$$

Then we obtain a differential inequality for $y_n(\tau)$, i.e.

$$\frac{dy_n(\tau)}{d\tau} \le M\cdot y_n(\tau) + F_n(\tau)$$

from which, in turn, by virtue of the theorem on differential inequalities, the inequality follows [17-19]

$$\frac{dy_n(\tau)}{d\tau} \le e^{G_1\tau}\cdot F(\tau)$$

Hence, we have:

$$\|\tilde{U}_n(r,\tau) - U_n(x,\tau)\|_2^2 \le p_0\|\varepsilon_0\|^2 + p_1\|\varepsilon_n\|^2 + p_2\|\Gamma_n^0\|^2 + p_3\|\Gamma_n'\|^2 + p_4\|\Gamma_n'\|^2 \qquad (14)$$

Where constants $p_i\left(i=\overline{0,4}\right)$ do not depends on N. Consequently,

$$\|\tilde{G}_n(\tau) - G_n(\tau)\|_{E_n}^2 \le \frac{1}{q}\|\tilde{U}_n(r,\tau)-U_n(r,\tau)\|_2^2 \le \frac{1}{q}\omega^2$$

where ω^2- is the right side of inequality (14). The last relations imply the stability of the algorithm for constructing an approximate solution and the numerical stability of the approximate solution in

CONCLUSION. An approximate solution of the Galerkin method is constructed for one boundary value problem of equations of parabolic type. The stability of the Galerkin method for the approximate solution of the problem under consideration is established under the condition of strongly minimal coordinate systems.

References:

1. Mamatov A., Parpiev A., Shorakhmedova, M. Mathematical model for calculating the temperature field of a direct-flow drying drum. Journal of Physics: Conference Seriesthis link is disabled, 2021, 2131(5), 052067

2.Mamatov, A., Bakhramov S., Narmamatov A. An approximate solution by the Galerkin method of a quasilinear equation with a boundary condition containing the time derivative of the unknown function. AIP Conference Proceedingsthis link is disabled, 2021, 2365, 070003

3. Mamatov A.Z., Usmankulov A.K., Abbazov I.Z., Norboyev U.A., Mukhametshina E.T. Determination of Temperature of Components of Cotton-Raw Material in a Drum Dryer with a Constant. IOP Conference Series: Earth and Environmental Sciencethis link is disabled, 2021, 939(1), 012052

4.Mamatov A.Z., Pardaev X.N., Mardonov, J.S.H., Plekhanov, A.F. Determining of the heat-moisture stateof raw cotton in a drum dryer. Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil'noi Promyshlennostithis link is disabled, 2021, 391(1), C. 46–49

5.Mamatov A., Parpiyev A., Kayumov A., Mathematical models of the heat and mass exchange process during pneumo-transportation of cotton-raw// International Scientific Journal Theoretical & Applied Science , p-ISSN: 2308-4944 (print) e-ISSN: 2409-0085 (online) , Year: 2020 Issue: 11 ,Volume: 91 P.508-513.

6.Mamatov A.Z., Axmatov N. Chislennoe reshenie zadachi opredeleniya teplo-vlazhnogo sostoyaniya xlopka-syrtsa v barabannoy sushilke // J.To'qimachilik muammolari.-2016.-№3,-B.80-86.

7.Mamatov A.Z., Djumabaev G. Ob ustoychivosti priblijennogo resheniya odnoy zadache opredeleniya teplo-vlazhnogo sostoyaniya xlopka-syrtsa //
J. Problemy tekstilya. 2010.-№2, P. 86-90

8.Mixlin S.G. Chislennaya realizatsiya variatsionnyx metodov. M.-Nauka,-1966.-432 Pages.

9.Mamatov A.Z. Primeneniya metoda Galerkina k nekotoromu kvazilineynomu uravneniyu parabolicheskogo tipa // Vestnik LGU,-1981.-№13.-P.37-45.

10. Mamatov A.Z., Baxramov S. Priblizhennoe reshenie metoda Galerkina kvazilineynogo uravneniya s granichno'm usloviem, soderjahiy proizvodnuyu po vremeni ot iskomoy funktsi // Uzbekistan -Malaysia inter. scientific online conference, Uz.NU, 24-25 august 2020 y., 239 p.

11.Mamatov A.Z., Dosanov M.S., Raxmanov J., Turdibaev D.X. Odna zadacha parabolicheskogo tipa s divergentnoy glavnoy chastyu // NAU (Natsionalnaya assotsiatsiya uchenyx). Monthly scientific journal, 2020, №57, 1-part, P.59-63.

12.
Mamatov, A.Z., Narjigitov X., Kengash J ., Rakhmanov J., Stability of the Galerkin Method for one Quasilinear Parabolic Type Problem// CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES 2 (6), 6-12, 2021

13.Mamatov, A.Z., Narjigitov X., Rakhmanov J., Turdibayev D. Refining the Galerkin method error estimation for parabolic type problem with a boundary condition E3S Web of Conferences 304, 03019 (2021) ICECAE 2021 https://doi.org/10.1051/e3sconf/202130403019

14. Ladyzhenskaya O.A., Solonnikov V.A., Uraltseva N.N. Lineyniye i kvazilineyniye uravneniya parabolicheskogo tipa. M.-Nauka,-1967.-736 P.

15. Ladyzhenskaya O. A., Uraltseva N.N. Lineyniye i kvazilineynie uravneniya ellipticheskogo tipa. M.-Наука,-1973.-576 C.

16. Wheeler M.F. A priori error estimates for Galerkin approximation to parabolic partial differential equations. SIAM J. Numer. Anal.1973.-10.-P.723-759.