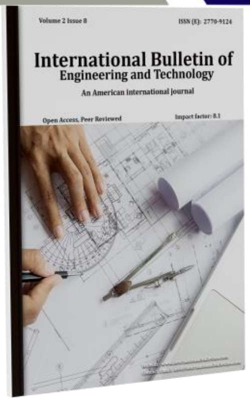

ISOMORPHISMS OF LOG-ALGEBRAS

Egamov Sevinchbek Maxsud o'g 'li
"Ma'mun-University" Non-GEI, teacher of the economy and accounting department.
E-mail: sevinchbek.private@mail.ru
Rahimov Mirjalol Sultonboy o 'g 'li
"Ma'mun-University" Non-GEI, student of economics.
E-mail: rahimovmirjalol0213@gmail.com
https://doi.org/10.5281/zenodo.7949777

Annatation: Let (Ω,μ) be a σ-finite measure space, and let $L_0(\Omega,\mu)$ be the *-algebra of all complex (real) valued measurable funktions on (Ω,μ). The *- subalgebra $L_{\log}(\Omega,\mu)=\left\{ f\in L_0(\Omega,\mu): \int_\Omega \log(1+|f|d\mu<+\infty)\right\}$ of $L_0(\Omega,\mu)$ is called the algebra of $\log-$ integrable measurable functions on (Ω,μ). Using the notion of passport of a normed Boolean algebra, we give the necessary and suffient conditions for a *-isomorphizm of two algebras of log-integrable measurable function.

Keywords: Integrable function,space, passport of Boolean algebra, Isomorphizm of log-algebras, log-integrable function

The study of L_p spaces was like Banach, who described the isometries of $L_p[0,1], p\neq 2$ spaces. The products of this direction were given by Yedon, who described all their isometries in L_p dimensions in different dimensions.

We denote the set of $\nabla=\nabla_\mu$ -dimensional functions as $L_0(\nabla_\mu)=L_0(\Omega,A,\mu)$.

Algebra $L_0(\nabla_\mu)$ was seen in the work "Space isometries of logarithm-integrable functions":

$$L_{\log}(\nabla_\mu)=f\in L_{\log}(\nabla_\mu): \|f\|_{\log}=\int_\Omega \log(1+|f|)d\mu\ <+\infty$$

$\log$ -integrable dimensional functions and every $f\in L_{\log}(\nabla_\mu)$ for

let's match $\|f\|_{\log}=\int_\Omega \log(1+|f|)d\mu$

$\|\cdot\|_{\log}: L_{\log}(\nabla_\mu)\to[0,\infty)$ functions are considered F-norm and satisfy the following conditions.

Explanation.

(i). $\|f\|_{\log}>0$ all of $0\neq f\in L_{\log}(\nabla_\mu)$;

(ii). $\|\alpha f\|_{\log,\mu}\leq\|f\|_{\log,\mu}$ any $f\in L_{\log}(\nabla,\mu)$ va $\alpha\leq 1$ numbers;

(iii). $\lim_{\alpha\to 0}\|\alpha f\|_{\log,\mu}=0$ all $f\in L_{\log}(\nabla_\mu)$ for;

(iv). $\|f+g\|_{\log,\mu}\leq\|f\|_{\log,\mu}+\|g\|_{\log,\mu}$ all of $f,g\in L_{\log}(\nabla_\mu)$ for.

Theorem.1. $L_{\log}(\nabla_\mu)$ and $L_{\log}(\nabla_\nu)$ F-spaces should be only this equation $\mu(\Omega)=\nu(\Omega)$.

We say that μ and ν (Ω, A) on measurable spaces and $0\le \frac{d\nu}{d\mu}:=h$ – Rodon-Nikodim's derivative, $\nu(x)=\mu(hx)$. That is clear, space

$$L_{\log}(\nabla_\nu)=\left\{f\in L_0(\nabla):\int_\Omega \log(1+|f|)d\nu<+\infty\right\}$$

$$\left\{f\in L_0(\nabla):\int_\Omega h\cdot\log(1+|f|)d\mu<+\infty\right\}=L^\nu{}_{\log}(\nabla_\mu)$$

and has the following norm

$$\|f\|_{\log,\nu}=\int_\Omega \log(1+|f|)d\nu=\int_\Omega h(\log(1+|f|))d\mu:\|f\|^\nu_{\log,\mu}.$$

We also need the following analogue of the space of log-integrable dimensional functions:

$$L^{(\nu)}{}_{\log}(\nabla_\mu)=\left\{f\in L_0(\nabla):\int_\Omega \log(1+h|f|)d\mu<+\infty\right\}\quad \|f\|^{(\nu)}_{\log,\mu}=\int_\Omega \log(1+h|f|)d\mu$$

F-norm. This follows from the following.

Explanation.

$\|f\|^{(\nu)}_{\log,\mu}$ function satisfies the following conditions:

(i). $\|f\|^{(\nu)}_{\log,\mu}>0$ all of $0\neq f\in L_{\log}(\nabla_\mu)$;

(ii). $\|\alpha f\|^{(\nu)}_{\log,\mu}\le\|f\|^{(\nu)}_{\log,\mu}$ any for $f\in L_{\log}(\nabla,\mu)$ and $\alpha\le 1$;

(iii). $\lim_{\alpha\to 0}\|\alpha f\|^{(\nu)}_{\log,\mu}=0$ all of $f\in L_{\log}(\nabla,\mu)$ for ;

(iv). $\|f+g\|^{(\nu)}_{\log,\mu}\le\|f\|^{(\nu)}_{\log,\mu}+\|g\|^{(\nu)}_{\log,\mu}$ all $g,\ f\in L_{\log}(\nabla,\mu)$ for.

Let's remind,

$$L_p(\nabla)=\left\{f\in L_0(\nabla):\int_\Omega |f|^p d\mu<+\infty\right\},\ \|f\|_{p,\mu}=(\int_\Omega |f|^p d\mu)^{\frac{1}{p}}.$$

$$L_p(\nabla_\mu)=\left\{f\in L_0(\nabla):\int_\Omega h|f|^p d\mu<+\infty\right\}$$

$$\|f\|_{p,\nu}=(\int_\Omega |f|^p d\nu)^{\frac{1}{p}}=(\int_\Omega (h|f|^p)d\mu)^{\frac{1}{p}}$$

Therefore, in the equivalent cases μ and ν $U:L_p(\nabla_\mu)\to L_p(\nabla_\nu)$ defined by the equation $U(f)=h^{-1}f$, $f\in L_p(\nabla_\mu)$ the opposite is a linear surjective isometry from $L_p(\nabla_\mu)$ to $L_p(\nabla_\nu)$.

Description. $L_{\log}^{\nu}(\nabla_{\mu})$ is an external log-space and $L_{\log}^{(\nu)}(\nabla_{\mu})$ is an internal log-space is called.The space $L_{\log}^{(\nu)}(\nabla_{\mu})$ is not an algebra at all.

Description. Meras μ and ν are called α-equivalent, if on ∇

$$L_{\log}^{\nu}\left(\nabla_{\mu}\right)=L_{\log}^{(\nu\cdot\alpha^{-1})}\left(\nabla_{\mu}\right).$$

if there is an automorphism satisfying the equality.

Theorem. If $L_{\log}^{\nu}(\nabla_{\mu})$ is the algebra $L_{\log}(\nabla_{\mu})$ and $L_{\log}^{(\nu)}(\nabla_{\mu})$, it is necessary and sufficient for the algebras to be isomorphic that μ and ν are α-equivalent.

Theorem. If ∇ is homogeneous, $L_{\log}(\nabla_{\mu})$ and $L_{\log}^{(\nu)}(\nabla_{\mu})$ are finite in dimension, and the algebras are isomorphic.

References:

1.Dykema K., Sukochev F., Zanin D. Algebras of log-integrable functions and operators. // Journal Complex Anal. Oper. Theory 10, 8. 2016. –С. 1775–1787.

2.Segal I., A non-commutative extension of abstract integration. // Ann. of Math. 1953 –С. 401–457.

3.Трунов Н.В. К теории нормальных весов на алгебрах Неймана. // Изв. вузов. Матем., 1982, 8, –С. 61-70.

4.Трунов Н.В. Lp-пространства, ассоциированные с весом на полуконечных алгебрах фон Неймана. // Констр. Теор. Функ. Анал. 3, 1981. –С.88-93.