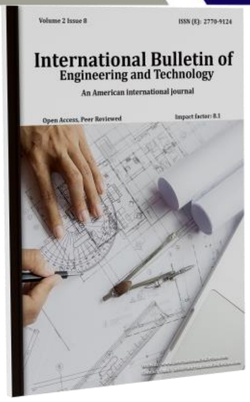


# STAGNATION IN THE RITZ AND GALYORKIN METHODS.

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**Abstract:** Ritz and Galerkin methods play an important role in variational and auxiliary methods. This is the case in Ritz and Galyorkin's articles on defining stagnation conditions with the help of examples. Ritz and Galyorkin prove that the project is equal to each other. This article examines the stationarity of the Ritz and Galyorkin methods for the Poisson's equation.
**Key words:** Poisson's equation, Ritz method, variational calculus, minimizing, *Matlab*, integrate over, Galyorkin method.

**INTRODUCTION**

The first problems related to calculus of variations arose in the 17th century, and in the process of solving such problems, this branch of mathematics developed.

Among the "historical" problems of calculus of variations are finding the path of propagation of light in a non-homogeneous medium (P. Fermat) and what shape should be in order for a body moving in a circular motion along an axis to meet the least resistance (I .Newton) issues are included.

Despite the fact that the above-mentioned problems were mainly solved in the 18th century, calculus of variations was formed as an independent science only thanks to the works of L. Euler.

The simplest problem of calculus of variations

$$\mathrm{F}(u)=\int_a^b \Phi\left(x,u,u'\right)dx \qquad (1.1)$$

is to find a function that gives the smallest value to the functional is piecewise differentiable $u(a)=A,\quad u(b)=B$ satisfies the conditions $u(x)$ where is a function $\Phi$ that is continuous differentiable with respect to its arguments.

**ANALYSIS AND RESULTS**

Poisson's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=q(x,y),\ 0<x<a, 0<y<b \qquad (1)$$

given It is below

$$u(0,y)=u(a,y)=u(x,0)=u(x,b)=0 \qquad (2)$$

we solve with the Ritz method in the boundary condition. It is known from the general theory of variational calculus that the following functional corresponds to the equation (1).

$$F(u)=\int_0^a\int_0^b\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2-2qu\right\}dxdy \quad (3)$$

The solution of the equation based on the Ritz method

$$u=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{i,j}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \quad (4)$$

Looking in the form, the boundary condition is satisfied and (2) takes the following form:

$$F(u)=\frac{\pi^2ab}{4}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left\{\left[\left(\frac{i}{a}\right)^2+\left(\frac{j}{b}\right)^2\right]a^2_{i,j}-2a_{i,j}\int_0^a\int_0^b q(x,y)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}dxdy\right\} \quad (5)$$

$a_{i,j}$ coefficients (5) from the condition of minimizing the functional, i.e

$$\frac{\partial F}{\partial a_{i,j}}=0 \qquad i,j=1,2,3,\ldots.. \quad (6)$$

is found from the condition. Based on (5) and (6), $a_{i,j}$ we create the following formula to find .

$$a_{i,j}=\frac{4}{\pi^2ab\left[\left(\frac{i}{a}\right)^2\left(\frac{j}{b}\right)^2\right]}\int_0^a\int_0^b q(x,y)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}dxdy \quad (7)$$

found $a_{i,j}$ ones in (4), then the solution (4) is the solution of equation (1) found by the Ritz method satisfying the boundary conditions (2).

Now we see that equation (1) is solved by the Galyorkin method under the boundary conditions (2). If we put the expression (4) in (1), then $\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$ multiply by and integrate over the area under consideration, we get the following equation:

$$u=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}A_{n,m,i,j}a_{i,j}=B_{n,m} \quad n,m=1,2,3,\ldots. \quad (8)$$

here

$$A_{n,m,i,j}=\int_0^a\int_0^b\left[\left(\frac{i}{a}\right)^2+\left(\frac{j}{b}\right)^2\right]\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}dxdy$$

$$B_{n,m}=\int_0^a\int_0^b q(x,y)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}dxdy \quad (9)$$

By performing the integration process, $a_{i,j}$ we again form the formula for the unknown coefficients we are looking for from . So, for the example under consideration, both methods give the same solution.

**Example 1.**

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=2x+y,$$

$$0<x<1, 0<y<0.5$$

$$u(0,y)=u(1,y)=u(x,0)=u(x,0,5)=0$$

given

Solving the problem in the Ritz and Galyorkin methods and checking the stability.

**MATLAB CODE OF THE PROBLEM:**

***In the Galyorkin method:***

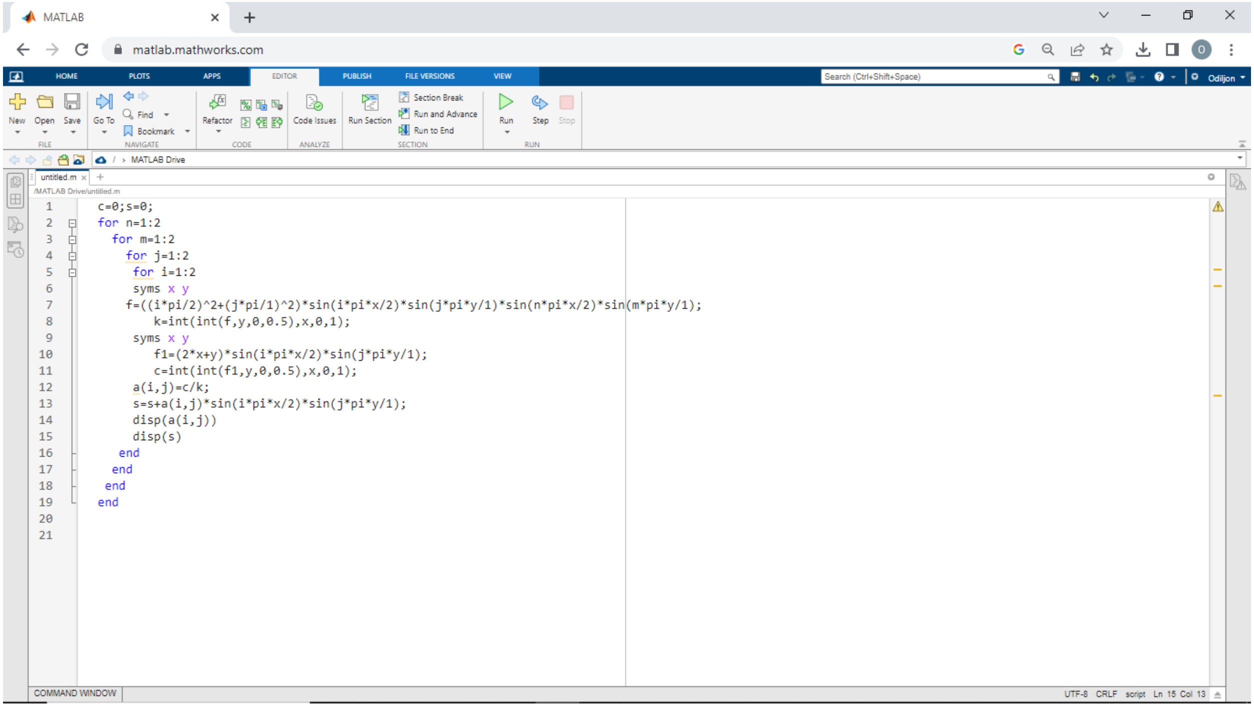
```
c=0;s=0;
for n=1:2
for m=1:2
for j=1:2
for i=1:2
syms x y
f=((i*pi/2)^2+(j*pi/1)^2)*sin(i*pi*x/2)*sin(j*pi*y/1)*sin(n*pi*x/2)*sin(m*pi*y/1);
k=int(int(f,y,0,0.5),x,0,1);
syms x y
f1=(2*x+y)*sin(i*pi*x/2)*sin(j*pi*y/1);
c=int(int(f1,y,0,0.5),x,0,1);
a(i,j)=c/k;
s=s+a(i,j)*sin(i*pi*x/2)*sin(j*pi*y/1);
disp(a(i,j))
disp(s)
end
end
end
end
```

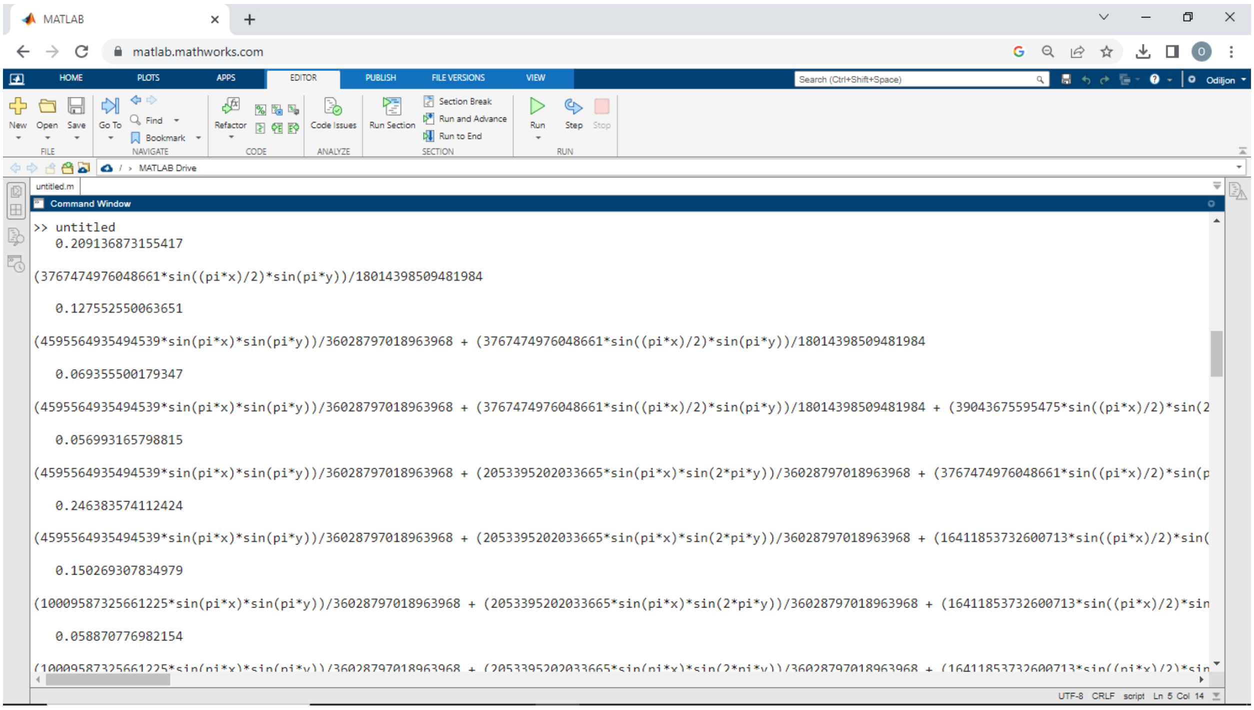
***In the Ritz method***

```
b=0;c=0;s=0;
for j=1:3
for i=1:3
b=4/(pi^2*2*1*((i/2)^2*(j/1)^2));
syms x y
f=(2*x+y)*sin(i*pi*x/2)*sin(j*pi*y/1);
c=int(int(f,y,0, 0.5), x, 0, 1);
a(i,j)=b*c;
disp(a(i,j));
s=s+a(i,j)*sin(i*pi*x/2)*sin(j*pi*y/1);
disp(s)
end
end
```

**RESULTS:**

*Galyorkin method:*


MATLAB
matlab.mathworks.com
c=0;s=0;
for n=1:2
for m=1:2
for j=1:2
for i=1:2
syms x y
f=((i*pi/2)^2+(j*pi/1)^2)*sin(i*pi*x/2)*sin(j*pi*y/1)*sin(n*pi*x/2)*sin(m*pi*y/1);
k=int(int(f,y,0,0.5),x,0,1);
syms x y
f1=(2*x+y)*sin(i*pi*x/2)*sin(j*pi*y/1);
c=int(int(f1,y,0,0.5),x,0,1);
a(i,j)=c/k;
s=s+a(i,j)*sin(i*pi*x/2)*sin(j*pi*y/1);
disp(a(i,j))
disp(s)
end
end
end
end


MATLAB
matlab.mathworks.com
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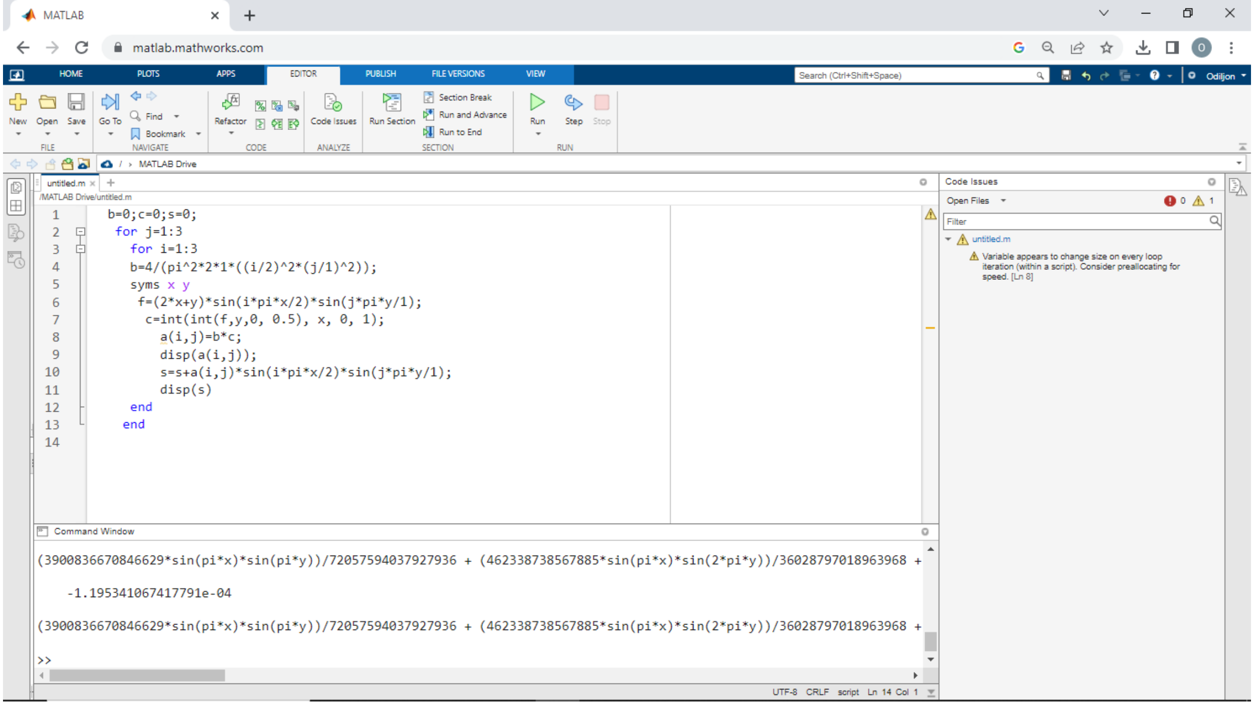
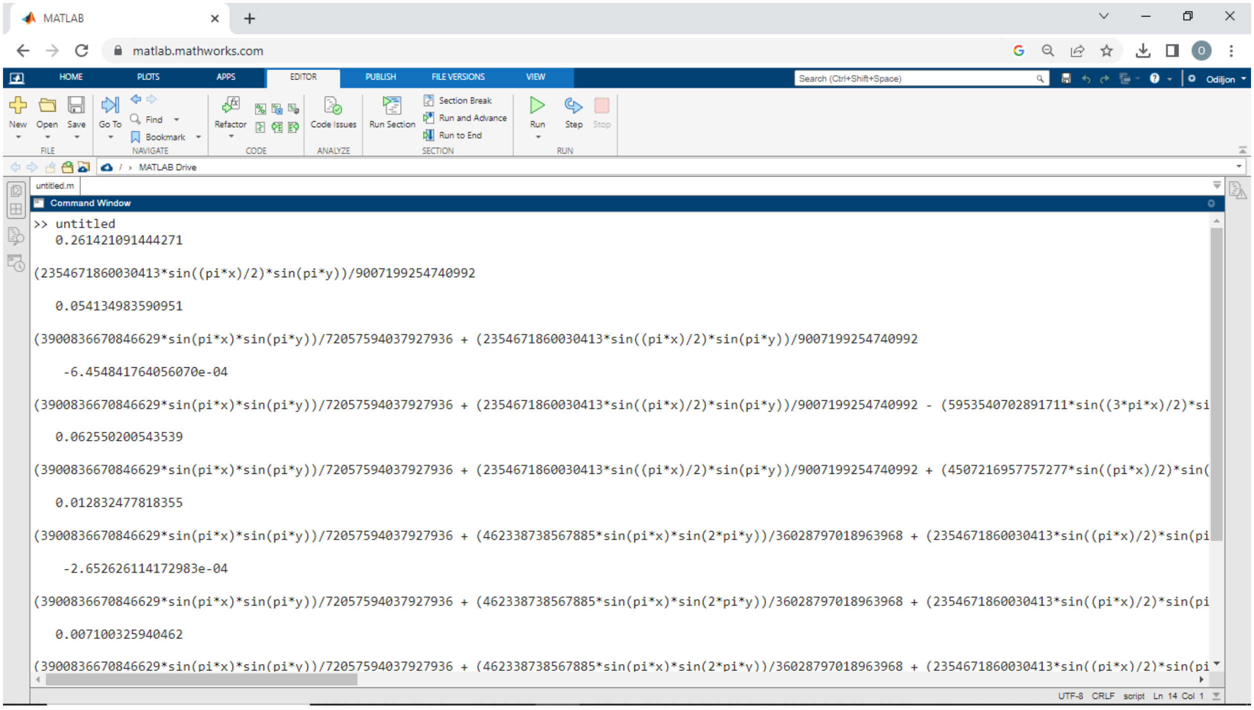
0.209136873155417
(3767474976048661*sin((pi*x)/2)*sin(pi*y))/18014398509481984
0.127552550063651
(4595564935494539*sin(pi*x)*sin(pi*y))/36028797018963968 + (3767474976048661*sin((pi*x)/2)*sin(pi*y))/18014398509481984
0.069355500179347
0.056993165798815
0.246383574112424
0.150269307834979
0.058870776982154

*Ritz Method:*

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