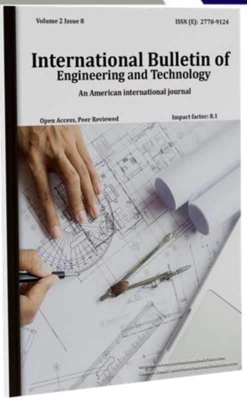

BOUNDARY VALUE PROBLEMS FOR LAPLACE EQUATIONS WITH VARIABLE ARGUMENTS

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Modern technologies related to mathematical modeling, as well as natural science achievements, make it necessary to search for new boundary value problems for any qualitative classes of differential equations. One such problem, along with ordinary differential equations, are boundary value problems for ordinary differential equations, and then a series of sets for such problems can be created [1-3].

$$u_{xx}(x,-y,-z)+u_{yy}(-x,y,-z)+u_{zz}(-x,-y,z)=0 \qquad (1)$$

the equation

$$\begin{cases} u(p,y,z)=u(-p,y,z)=0, \\ u(x,q,z)=u(x,-q,z)=0, \end{cases} \qquad (2)$$

by the boundary conditions and unity

$$u(x,y,r)=u(x,y,-r)=\varphi(x,y) \qquad (3)$$

$$u(x,y,r)=-u(x,y,-r)=\psi(x,y) \qquad (4)$$

the question of finding a solution that satisfies one of the boundary conditions is considered, there $\varphi(x,y)$ and $\psi(x,y)$ functions $[-p,p]\times[-q,q]$ functions with no uncertainty, as defined by a regular parallelogram.

In this article, if $u(x,y,z)$ function z If there is an even function in the argument, then (1),(2) is a single-boundary problem satisfying the boundary condition (3), and if there is an even function, then (1),(2) is a single-boundary problem satisfying the boundary condition (4) the problem of finding is considered. Firstly we can find the solution of the boundary value problem (1) - (3)

$$u(x,y,z)=X(x)Y(y)Z(z) \qquad (5)$$

Search like that, unknown source $X(x),Y(y)$ and $Z(z)$ we put the determinant of the function in equation (5) to (1)

$$X''(x)Y(-y)Z(-z)+X(-x)Y''(y)Z(-z)+X(-x)Y(-y)Z''(z)=0$$

Two sides of this equation $X(-x)Y(-y)Z(-z)$ divided by the exponent:

$$\frac{X''(x)}{X(-x)}+\frac{Y''(y)}{Y(-y)}+\frac{Z''(z)}{Z(-z)}=0.$$

corresponding to the first and second joiners on the left side of the last equation $-\lambda_1^2$ and $-\lambda_2^2$ equate. There the last three digits are joined $\lambda_1^2+\lambda_2^2$ and equal to this, so

$$\frac{X''(x)}{X(-x)} = -\lambda_1^2, \quad \frac{Y''(y)}{Y(-y)} = \lambda_2^2, \quad \frac{Z''(z)}{Z(-z)} = \lambda_1^2 + \lambda_2^2.$$

We're going to put (5) and (2) on the boundary conditions, $X(x)$ and $Y(y)$ to functions

$$X''(x) + \lambda_1^2 X(-x) = 0, \quad X(-p) = X(p) = 0,$$
$$Y''(y) + \lambda_2^2 Y(-y) = 0, \quad Y(-q) = Y(q) = 0, \qquad (6)$$

We have the Sturm-Lewville problems in Turing.

As can be seen from the boundary conditions in (6), if $X(x)$ and $Y(y)$ If the functions are odd functions, then (6) the boundary problems have only a null solution. And if $X(x)$ and $Y(y)$ if the function is an even function, so (6) If we want to obtain the first equation of 6, the solution to this constrained problem will not be zero $X(x)$ when the function is an even function

$$X''(x) + \lambda_1^2 X(x) = 0, \quad X(p) = 0$$

Like this

$$X(x) = c_1 \sin \lambda_1 x + c_2 \cos \lambda_1 x$$

This function will be a pair of functions $c_1 = 0$ In this case, the solution to the equation is

$$X(x) = c_2 \cos \lambda_1 x$$

to have the opportunity to satisfy the conditions of this decision $X(p) = 0$

$$X(p) = c_2 \cos \lambda_1 p = 0$$

Also $c_2 = 1$ equal to

$$\cos \lambda_1 p = 0$$

or

$$\lambda_1 p = \frac{\pi}{2} + \pi m$$

To λ_1

$$\lambda_{1m} = \frac{\pi(2m+1)}{2p}, \quad m = 0,1,2,\ldots$$

and $X(x)$ function equality

$$X_m(x) = \cos \frac{\pi(2m+1)}{2p} x, \quad m = 0,1,2,\ldots$$

Have the meaning like this.

$X(x)$ If we apply this method of determining the function to the secondary boundary problem in (6), and $Y(y)$.

$$Y_n(y) = \cos \frac{\pi(2n+1)}{2q} y, \quad \lambda_{2n} = \frac{\pi(2n+1)}{2q}, \quad n = 0,1,2,\ldots$$

So $Z(z)$ function equal to

$$Z''(z)-(\lambda_{1m}^2+\lambda_{2n}^2)Z(-z)=0 \qquad (7)$$

(3) as seen from the boundary condition $Z(z)$ The function of a pair of functions in a coordinate system (7) equality

$$Z''_{mn}(z)-(\lambda_{1m}^2+\lambda_{2n}^2)Z_{mn}(z)=0$$

Solution to this equation

$$Z_{mn}(z)=A_{mn}e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}+B_{mn}e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}$$

By Condition $Z(z)$ when the function is $A_{mn}=B_{mn}=C_{mn}$ and $Z(z)$ to this function

$$Z_{mn}(z)=C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}\right)$$

So (1) equality (2) Decide whether to satisfy the limit conditions

$$u(x,y,z)=\sum_{m,n=0}^{\infty}C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}\right)\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y \qquad (8)$$

In this place C_{mn} There are an unknown number of coefficients that need to be determined . We can take the determinant of these unknown coefficients to the boundary condition (8) to (3):

$$\sum_{m,n=0}^{\infty}C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}\right)\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y=\varphi(x,y). \qquad (9)$$

So C_{mn} coefficient

$$C_{mn}==\frac{4}{pq\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}\right)}\int_{-p}^{p}\int_{-q}^{q}\varphi(x,y)\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}ydxdy$$

Will be like this .

If $\varphi(x,y)$ this function

$$\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y$$

by mechanical functions

$$\varphi(x,y)=\sum_{m,n=0}^{\infty}\varphi_{mn}\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y$$

From (9) to C_{mn} coefficient function

$$C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}\right)=\varphi_{mn}$$

From this

$$C_{mn}=\frac{\varphi_{mn}}{e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}}$$

In this place

$$\varphi_{mn}=\frac{4}{pq}\int_{-p}^{p}\int_{-q}^{q}\varphi(x,y)\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}ydxdy.$$

So (1),(2) Consider the problem of finding a solution to the boundary problem that satisfies the boundary condition (4). In order to this $Z(z)$ assume that this is an odd function. So (5) equation

$$Z''_{mn}(z)+(\lambda_{1m}^2+\lambda_{2n}^2)Z_{mn}(z)=0$$

This equation can be written in the following form:

$$Z_{mn}(z)=A_{mn}\cos\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z+B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z$$

And $Z(z)$ function $A_{mn}=0$ and $Z(z)$ to this function

$$Z_{mn}(z)=B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z$$

We have the equation like this .

And (1) equation (4) deciding whether to satisfy the threshold condition

$$u(x,y,z)=\sum_{m,n=0}^{\infty}B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y \qquad (10)$$

Unknown B_{mn} to identify the coefficient (10) to (4) :

$$\sum_{m,n=0}^{\infty}B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,r\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y=\psi(x,y). \qquad (11)$$

If $\psi(x,y)$ to function

$$\psi(x,y)=\sum_{m,n=0}^{\infty}\psi_{mn}\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y$$

And (11) to $B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,r=\psi_{mn}$ from this

$$B_{mn}=\frac{\psi_{mn}}{\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,r}$$

Is like that.

$$\psi_{mn}=\frac{4}{pq}\int_{-p}^{p}\int_{-q}^{q}\psi(x,y)\cos\frac{\pi(2m+1)}{2p}x\cos\frac{\pi(2n+1)}{2q}y\,dxdy.$$

For instance

$$u_{xx}(x,-y,-z)+u_{yy}(-x,y,-z)+u_{zz}(-x,-y,z)=0 \qquad (12)$$

equations

$$\begin{cases}u(\pi,y,z)=u(-\pi,y,z)=0,\\ u(x,\pi,z)=u(x,-\pi,z)=0,\end{cases} \qquad (13)$$

lower than the one in the border conditions

$$u(x,y,\pi)=u(x,y,-\pi)=(x+1)(y+1) \qquad (14)$$

If we consider the problem of finding a solution that satisfies the boundary charter, then according to solution (8)

$$u(x,y,z)=\sum_{m,n=0}^{\infty}C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z}\right)\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y \qquad (15)$$

In this place

$$\lambda_{1m}=\frac{2m+1}{2},\quad \lambda_{2n}=\frac{2n+1}{2}.$$

Unknown C_{mn} to identify the coefficient (14) we use a boundary condition:

$$\sum_{m,n=0}^{\infty} C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}\right)\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y=(x+1)(y+1),$$

To

$$C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}\right)=$$

$$=\frac{1}{\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}(x+1)(y+1)\cos\frac{2m+1}{2}x\cos\frac{2n+1)}{2}ydxdy=$$

$$=\frac{16(-1)^m(-1)^n}{\pi^2(2m+1)(2n+1)}$$

And

$$C_{mn}=\frac{16(-1)^m(-1)^n}{\pi^2(2m+1)(2n+1)}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}\right)^{-1}$$

So in this place C_{mn} the meaning of coefficients (15) in the same place as the one mentioned in the example (12)-(14) functions

$$u(x,y,z)=\sum_{m,n=0}^{\infty} C_{mn}\left(e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}\right)\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y=$$

$$=\frac{16}{\pi^2}\sum_{m,n=0}^{\infty}\frac{(-1)^m(-1)^n}{(2m+1)(2n+1)}\frac{e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}z}}{e^{\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}+e^{-\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}r}}\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y$$

If (12),(13) this function

$$u(x,y,\pi)=-u(x,y,-\pi)=(x+1)(y+1) \qquad (16)$$

If a solution is required, then such a solution is determined by formula (10), it

$$u(x,y,z)=\sum_{m,n=0}^{\infty} B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y$$

This (16) will equal to

$$\sum_{m,n=0}^{\infty} B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,\pi\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y=(x+1)(y+1)$$

or

$$B_{mn}=\frac{16(-1)^m(-1)^n}{\pi^2(2m+1)(2n+1)\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,\pi}$$

B_{mn} set this value to its current position (12),(13) this function (16) to

$$u(x,y,z)=\sum_{m,n=0}^{\infty} B_{mn}\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y=$$

$$=\frac{16}{\pi^2}\sum_{m,n=0}^{\infty}\frac{(-1)^m(-1)^n\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,z}{(2m+1)(2n+1)\sin\sqrt{\lambda_{1m}^2+\lambda_{2n}^2}\,\pi}\cos\frac{2m+1}{2}x\cos\frac{2n+1}{2}y$$

This equation.

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